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Nonlinear Dynamic Analysis of Laminated Composite Plates Supported by Springs

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Abstract

In this study, nonlinear dynamic analysis of laminated composite plates supported by springs is investigated. Three different plate materials are selected for the analysis. Nonlinear free vibrations of laminated composite plates are studied in two different boundary conditions. These boundary conditions are free and simply supported. The classical finite element method is used for the analysis. Spring supports stiffness matrix and nonlinear stiffness matrix are developed specifically to analysis. A code of computer program with Matlab is prepared for the calculations. Effect of boundary conditions and material properties are investigated on nonlinear free vibrations of composite plate. The results are graphically presented and discussed.

Key words: Nonlinear dynamic analysis, laminated composite plates, finite element method, Geometric nonlinearity

1. Introduction

Dynamic analysis of composite plates is an important issue. This issue will continue to attract the attention of many researchers. In the literature, dynamic analysis of composite plate is usually linear. Nonlinear analyzes has been neglected in terms of the difficulty of calculation. In the literature, dynamic analyzes of composite plate is usually linear. Nonlinear analyzes has been neglected in terms of the difficulty of calculation. Additionally, the researches on nonlinear dynamic analysis of composite plates of supported by springs are quite a few. In the designing of composite plates is very important to know frequency values of nonlinear. The nonlinear frequencies may be required in terms of optimum design. Shooshtari et al. investigated linear and nonlinear free vibrations of composite and fiber metal laminated plates [1]. Lal et al. studied nonlinear free vibration analysis of laminated composite on elastic foundation [2]. Singh et al. [3] presented post-buckling and nonlinear free vibration analysis of a laminated composite. Sobhy dealt with vibration and buckling behavior of exponentially graded material sandwich plate resting on elastic foundations under various boundary conditions [4]. Patel et al. investigated dynamic instability of laminated composite plates supported on elastic foundations [5]. Li determined the modal characteristics of a rectangular plate with general elastic supports alone its edges [6]. Ashour presented semi-analytical solutions to determine the natural frequencies and the mode shapes of angle-ply laminated plates with edges elastically restrained [7]. Malekzadeh et al. studied large deformation analysis of composite plates on nonlinear elastic foundations [8]. Khov et al. calculated the static deflections and modal characteristics of orthotropic plates with general elastic boundary supports [9]. Kucukrendeci et al. presented effect of elastic boundary

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conditions on free vibration of laminated composite plates [10]. In this study, nonlinear free vibration of laminated composite plates supported by springs is investigated. Spring supports stiffness matrix and nonlinear stiffness matrix are written specifically to analyzes. Effect of boundary conditions and material properties are studied on nonlinear free vibrations of composite plate. The results are graphically presented.

2. Finite Element Method

A rectangular element, which is under the effect of bending vibrations, is shown at Fig. 1. There are three degrees of freedom at each node, at each corner. There are three degrees of freedom at each node, respectively, deflection of z direction and the two rotations, w, $\theta_x = \partial w/\partial y$ and $\theta_y = -\partial w/\partial x$. In terms of then on-dimensional (ξ,η) coordinates, $\theta_x = \frac{1}{b}\frac{\partial w}{\partial \eta}, \theta_y = -\frac{1}{a}\frac{\partial w}{\partial \xi}$.

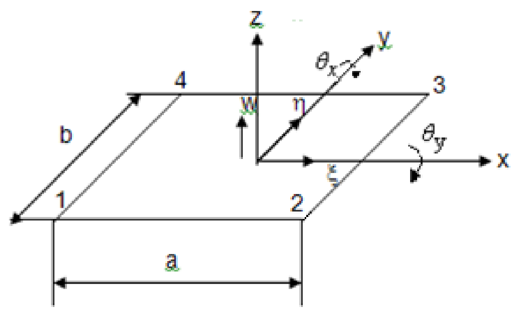

Figure 1. Geometry of a rectangular element ($\xi = \frac{x}{a}, \eta = \frac{y}{b}$).

Since the element has twelve degrees of freedom, the displacement function can be represented by a polynomial having twelve terms due to simplicity. It can be written as follows related to $\xi = \pm 1$ and $\eta = \pm 1$ coordinate, at the node points.

w=[$N_1(\xi,\eta)N_3(\xi,\eta)N_3(\xi,\eta)N_4(\xi,\eta)$] $\{w\}_e$

$$w=[N(\xi,\eta)]\{w\}_e \tag{1}$$

Where $\{w\}_e$ is the displacement and rotations vector

$$\{w\}_e^T = [w_1\, \theta_{x1}\, \theta_{y1} \ldots w_4\, \theta_{x4}\, \theta_{y4}] \tag{2}$$

At (2.1), defined the N (ξ,η) is

$$N_j^T(\xi,\eta) = \begin{bmatrix} (1/8)(1+\xi_j\xi)(1+\eta_j\eta)(2+\xi_j\xi+\eta_j\eta-\xi^2-\eta^2) \\ (b/8)(1+\xi_j\xi)(\eta_j+\eta)(\eta^2-1) \\ (a/8)(\xi_j+\xi)(\xi^2-1)(1+\eta_j\eta) \end{bmatrix} \tag{3}$$

and (ξ_j,η_j) are the coordinates of node j [11].

2.1. Mass matrix for plate element

The kinetic energy expressions for thin plate bending element is

$$T = \frac{1}{2}\int_A \rho h \dot{w}^2 dA \tag{4}$$

where is ρ density, h is thick plate and A is area of plate. Substituting Eq. (1) into Eq. (4) gives

$$T_e = \frac{1}{2}\{\dot{w}\}_e^T [M]_e \{\dot{w}\}_e \tag{5}$$

where

$$[M]_e = \int_{Ae} \rho h [N]^T [N] dA \quad \text{or} \quad [M]_e = \rho hab \int_{-1}^{+1}\int_{-1}^{+1} [N(\xi,\eta)]^T [N(\xi,\eta)] d\xi\, d\eta \tag{6}$$

Eq. (6) is element mass matrix. If $N_J(\xi,\eta)$ substitute from Eq. (3) and integrate equation Eq. (6), the result will be as follows [11]

$$[M]_e = \frac{\rho hab}{6300}\begin{bmatrix} m_{11} & m_{21}^T \\ m_{21} & m_{22} \end{bmatrix} \tag{7}$$

2.2. Linear stiffness matrix for composite plate element

The strain energy can be expressed in the [11]:

$$U_L = \frac{1}{2}\int_A \frac{h^3}{12}\{\chi\}^T [D]\{\chi\} dA \tag{8}$$

where

$$\{\chi\} = \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} & \frac{\partial^2 w}{\partial y^2} & 2\frac{\partial^2 w}{\partial xy} \end{bmatrix}. \tag{9}$$

and

$$D_{ij} = 1/3\sum_{k=1}^{N}(\overline{Q}_{ij})_k\,(z^3{}_k - z^3{}_{k-1}) \tag{10}$$

$\overline{Q}_{ij}$ a matrix of reduced stiffness components for the k[th] layer whose surfaces are at distances z_{k-1}, z_k from the middle surface of the plate. $\overline{Q}_{ij}$ are the components transformed lamina stiffness matrixes which are defined as follows;

$$\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}. \tag{11}$$

Terms of the $\overline{Q}_{ij}$ matrix are;

$$\begin{aligned}
&\overline{Q}_{11} = Q_{11}f^4 + 2(Q_{12}+2Q_{66})f^2g^2 + Q_{22}g^4, \qquad \overline{Q}_{12} = (Q_{11}+Q_{22}-4Q_{66})f^2g^2 + Q_{12}(f^4+g^4) \\
&\overline{Q}_{22} = Q_{11}g^4 + 2(Q_{12}+2Q_{66})f^2g^2 + Q_{22}f^4, \qquad \overline{Q}_{16} = (Q_{11}-Q_{12}-2Q_{66})f^3g + (Q_{12}-Q_{22}+2Q_{66})fg^3 \\
&\overline{Q}_{26} = (Q_{11}-Q_{12}-2Q_{66})fg^3 + (Q_{12}-Q_{22}+2Q_{66})f^3g, \quad \overline{Q}_{16} = (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})f^2g^2 + Q_{66}(f^4+g^4)
\end{aligned} \tag{12}$$

$f = cos\theta,\ g = sin\theta$,

$$Q_{11} = \frac{E_1}{1-\vartheta_{12}\vartheta_{21}},\ Q_{12} = \frac{\vartheta_{12}E_2}{1-\vartheta_{12}\vartheta_{21}} = \frac{\vartheta_{21}E_1}{1-\vartheta_{12}\vartheta_{21}},\ Q_{22} = \frac{E_2}{1-\vartheta_{12}\vartheta_{21}},\ Q_{66} = G_{12}$$

Substituting Eq. (8) into Eq. (1) gives

$U_e = \frac{1}{2}\{w\}_e^T[K]_e\{w\}_e$ (13)

$[K]_e$ can be written as follows;

$[K]_e = \int_A [B]^T [D][B] dA$ (14)

Equation (14) is the element stiffness matrix. And

$$[B] = \begin{bmatrix} \frac{1}{a^2}\frac{\partial^2}{\partial \xi^2} \\ \frac{1}{b^2}\frac{\partial^2}{\partial \eta^2} \\ \frac{2}{ab}\frac{\partial^2}{\partial \xi \partial \eta} \end{bmatrix} [N(\xi,\eta)] \quad (15)$$

dA = dxdy, $\xi = \frac{x}{a}, \eta = \frac{y}{b}, dA = abd\xi d\eta$ equal by is Eq. (14) new expression

$[K]_e = ab\int_A [B]^T [D][B] d\xi d\eta$ (16)

Element stiffness matrix terms are separated square underside matrix and stiffness matrix is symmetric [12].

$$[K]_e = \begin{bmatrix} [K]_{11} & & & sym. \\ [K]_{21} & [K]_{22} & & \\ [K]_{31} & [K]_{32} & [K]_{33} & \\ [K]_{41} & [K]_{42} & [K]_{43} & [K]_{44} \end{bmatrix} \quad (17)$$

2.3. Nonlinear stiffness matrix for composite plate element

The geometrical nonlinear strain energy for the high displacement systems can be given as

$U_{NL} = \frac{1}{2}\int_v \{\psi\}^T [\sigma_0]\{\psi\} dV$ (18)

where the matrix $[\sigma_0]$ is the axial stresses of the element calculated linearly. $\{\psi\}$ is the nonlinear displacement in the z direction. The terms of the matrix $[\sigma_0]$ can be found using

$\{\sigma_0\} = [D]\{\varepsilon\}$ (19)

The matrix [D] was defined previously in Eq. (10). Here, $\{\varepsilon\}$ represents the axial displacement on the other hand, $[\sigma_0]$ and $\{\psi\}$ are defined as

$$[\sigma_0] = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \quad (20)$$

and

$$\{\psi\} = \begin{bmatrix} \frac{\partial w}{\partial x} \\ \frac{\partial w}{\partial y} \end{bmatrix} . \quad (21)$$

Equation (18) can be re-defined using the thickness of the plate element, *h*, as

$U_{NL} = \frac{1}{2}\int_A h\{\psi\}^T [\sigma_0]\{\psi\} dA$ (22)

Substituting (1) in (22) gives

$$U_{eNL} = \frac{1}{2}\{w_*\}^T [K]_{eNL} \{w_*\} \tag{23}$$

Here, $\{w_*\}$ is the nonlinear displacement vector. $[K]_{eNL}$ matrix represents the nonlinear stiffness matrix of the element and can be defined with the Eq. (18) considering plate geometry, as

$$[K]_{eNL} = \int_A h\left[B'\right]^T [\sigma_0]\left[B'\right] dA \tag{24}$$

or

$$[K]_{eNL} = abh\int_{-1}^{+1}\int_{-1}^{+1}\left[B'\right]^T [\sigma_0]\left[B'\right] d\xi d\eta \tag{25}$$

Here, the matrix $\left[B'\right]$ can be defined using equation (3) as

$$\left[B'\right] = \begin{bmatrix} \frac{1}{a}\frac{\partial}{\partial\xi} \\ \frac{1}{b}\frac{\partial}{\partial\eta} \end{bmatrix} [N(\xi, \eta)] \tag{26}$$

The nonlinear stiffness matrix of the element can now be obtained using Eq. (25) in the form of 3x3 square partitioned sub-matrixes and can be given as

$$[K]_{eNL} = \begin{bmatrix} [K_*]_{11} & & & sym. \\ [K_*]_{21} & [K_*]_{22} & & \\ [K_*]_{31} & [K_*]_{32} & [K_*]_{33} & \\ [K_*]_{41} & [K_*]_{42} & [K_*]_{43} & [K_*]_{44} \end{bmatrix} \tag{27}$$

2.4 Analysis of linear undamped free vibration of laminated composite plates of supported by springs

Free vibration analysis of the laminated composite plates is made by eq. 28

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\} \tag{28}$$

where [M] and [K] are system mass matrix and system stiffness matrix respectively. System matrix is consists in combining element matrix (Eq. 7, 17).

$$([K] - \omega^2 [M])\{\Phi\} = \{0\} \quad . \tag{29}$$

In linear free vibration analysis of a structure, the solve of the linear eigenvector problem is necessary to determine the natural frequencies "ω" and modes of vibration [12]. The parameters of spring are adding to [K] matrix. New equation has been rewritten as Eq. 30, 31.

$$[M]\{\ddot{u}\} + ([K] + [K^s])\{u\} = \{0\} \tag{30}$$

$$[([K] + [K^s]) - \omega^2 [M]]\{\Phi\} = \{0\} \tag{31}$$

Where $[K^s]$ is parameters of spring.

2.5 Analysis of nonlinear undamped free vibration of laminated composite plates of supported by springs

The nonlinear free vibration analysis is carried out numerically by solving a geometrically nonlinear system with an appropriate iteration step for each deformation stage. The analysis of nonlinear free vibration requires a series of operations [13]. The nonlinear element stiffness matrix $[K]_{eNL}$ is given in Eq. 27. Nonlinear system stiffness matrix $[K]_{NL}$ consists of combining nonlinear element stiffness matrix.

$$[([K] + [K]_{NL} + [K^S]) - \omega_{NL}^2 [M]] \{\Phi_{NL}\} = \{0\} \tag{32}$$

In Eq. 33, [K] is the linear system stiffness matrix, [M] is the system mass matrix and $[K^S]$ is the spring matrix. The nonlinear natural frequencies (ω_{NL}) and nonlinear unit vector $\{\Phi_{NL}\}$ are calculated by solving Eq. 32. In the analyzes, the damping factor was not taken into account.

3. Physical Properties of the Composite Plates

In this study, three different plate models are selected. These models and boundary conditions are shown in Figure 2 and Figure 3. The value of the spring stiffness is selected as k=1250 N/m.

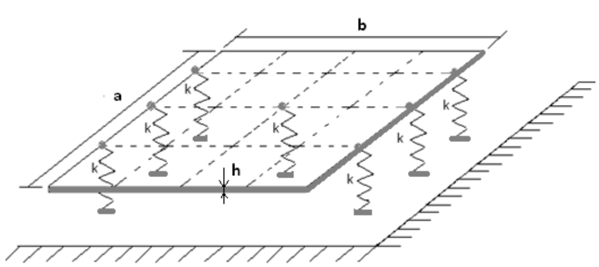

Figure 2. Free boundary conditions model of laminated composite plate of supported by springs

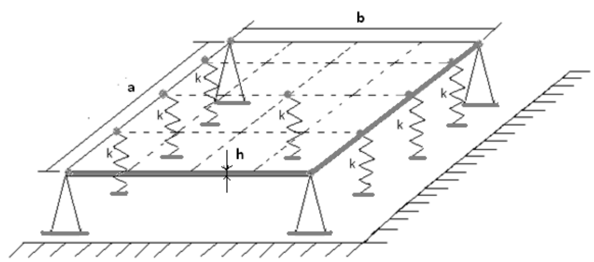

Figure 3. Simply supports boundary conditions model of laminated composite plate of supported by springs

Material properties of fiber fabrics are shown in Table 1. The thickness of fiber fabrics is chosen as 0.2 mm. Total plate thickness is h=1mm. A specific computer program is prepared with Matlab software for the calculations.

Table 1. Material properties of fiber fabrics [14, 15]. The thickness of fiber fabrics is 0.2 mm.

Type	ρ (kg/m^3)	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	ν_{12}	ν_{21}
AS/3501 Graphite/Epoxy	1630	138	9.0	6.9	0.3	0.019
Kev.49/934 Kevlar/Epoxy	1384	76	5.5	2.3	0.34	0.024
Boron/5505 Boron/Epoxy	2242	204	18.5	5.59	0.23	0.022

In this study, flat composite laminated plates are used. All plate models are in the form of rectangular. The dimensions of all the plate models are selected a=0.45 m and b=0.30 m. Composite plates were formed by five laminates of overlapping lined up. Laminates are lined up symmetrically (θ, -θ, θ, -θ, θ). θ is orientation angle of laminate. Two different orientation angle is selected for analyzes. These angles are 15^0 and 30^0. For example, $\theta = 15^0$, angles of five laminated plate are 15^0, -15^0, 15^0, -15^0, 15^0, respectively. The laminates are fiber fabrics. In this study, the material of fiber fabrics is selected AS/3501 graphite/epoxy, Kev.49/934 Kevlar/epoxy and Boron/5505 Boron/epoxy.

4. Numerical Analysis

The boundary conditions of composite plate given in Figure 4 are selected for linear analysis. The values of natural frequency of the undamped free vibration are calculated according the Eq. 29. The frequency parameter "λ" is obtained with the formula given in Eq. 33. In similar conditions, frequency parameters are found in [16, 17] for composite plates. Ref. 16 and 17 used different finite element method. In the first 14 vibration modes, frequency parameters are compared with Ref. 16-17 (Figure 5 and 6). Similar results are obtained.

$D_0=E_1h^3/12(1-\nu_{12}\,\nu_{21})$

$$\lambda = \rho h\omega^4 a^4/D_0 \qquad (33)$$

In this comparison, the material of composite plate is the graphite-epoxy. Plate boundary condition is fully clamped edges (Fig. 4).

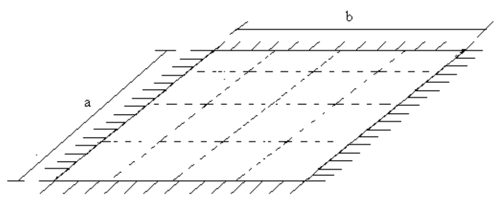

Figure 4. Fully clamped boundary conditions model of laminated composite plate (4x4 mesh element).

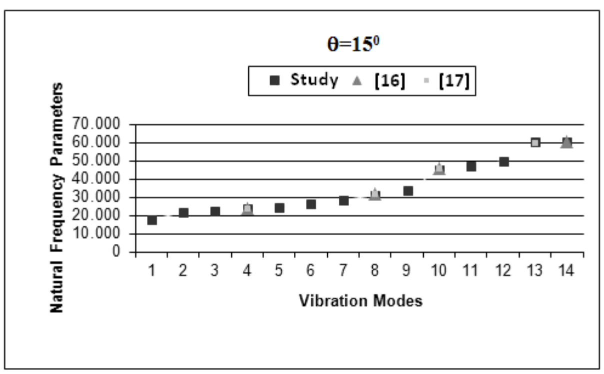

Figure 5. $\theta=15^0$, Linear natural frequency parameters of fully clamped boundary conditions model of laminated composite plate.

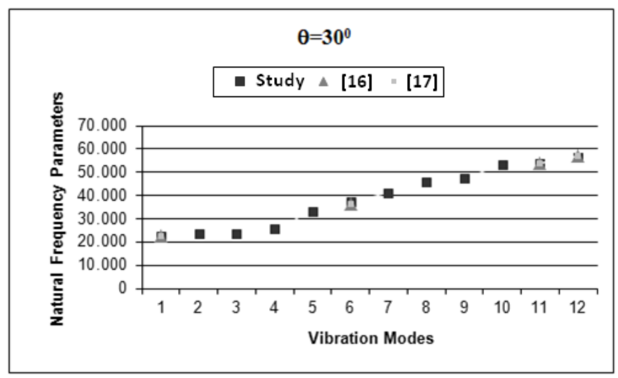

Figure 6. $\theta=30^0$, Linear natural frequency parameters of fully clamped boundary conditions model of laminated composite plate.

The plate dimensions are a=0.45 m and b=0.30 m. The material constants of the composite plate were modulus of elasticity E1=120 GPa and E_2=7.79 GPa, modulus of shear G_{12} = 6.15 GPa and poisons ratios ν_{12}=0.3 and ν_{21}= 0.019 [16]. In linear analyzes, orientation angles (θ) of laminate are15^0 and 30^0. In finite element method, the matrix [K], [M] (in Eq. 29) selected for calculation.

Nonlinear frequencies are calculated from Eq.32. Eq.33 is used for the calculation of nonlinear frequencies parameters. According to boundary conditions, spring matrix $[K^S]$ and nonlinear system stiffness matrix $[K]_{NL}$ is written specifically for this study. In nonlinear analyzes, the geometrical nonlinearity are used. In all the selected boundary conditions (Fig. 2, 3), the nonlinear frequency parameters of laminated composite plates (with orientation angles of $\theta=15^0$

and θ=30^0) are given graphically in Figures 7, 8, 9 and 10. In Figures 7, 8, 9 and 10 graphics show the frequency value for the first 25 modes.

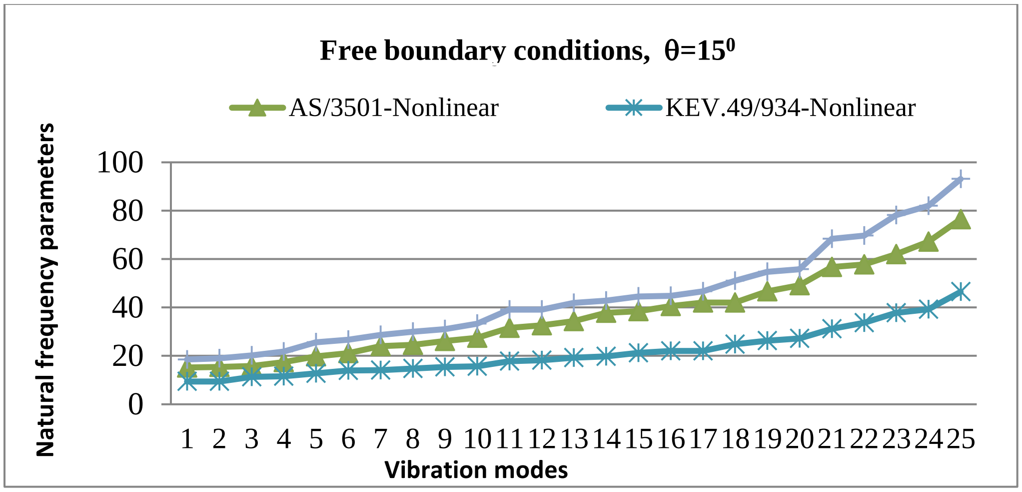

Figure 7. θ=15^0, AS/3501 Graphite/epoxy, Kev.49/934 Kevlar/epoxy, Boron/5505 Boron/epoxy, nonlinear frequency parameters of free boundary conditions, a=0.45 m and b=0.30 m.

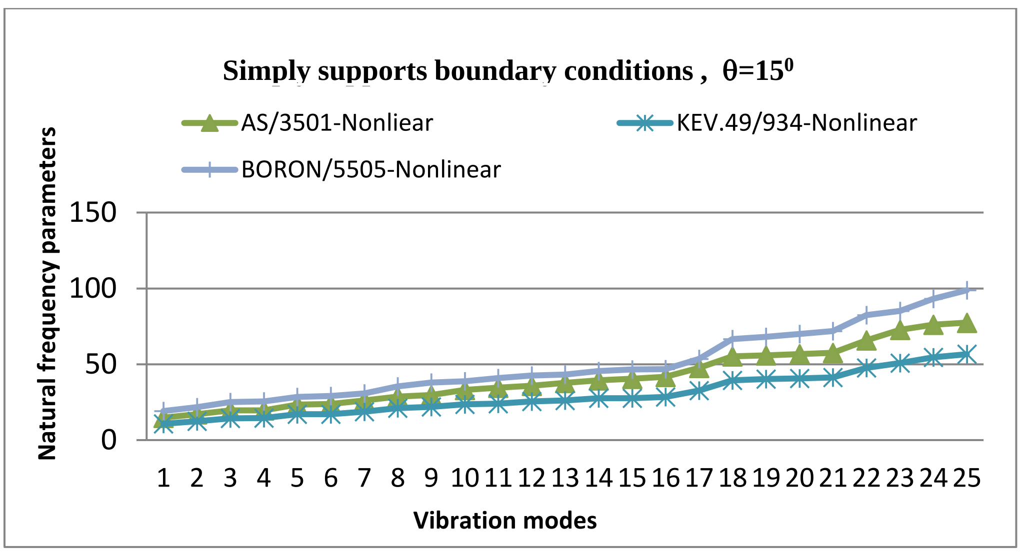

Figure 8. θ=15^0, AS/3501 Graphite/epoxy, Kev.49/934 Kevlar/epoxy, Boron/5505 Boron/epoxy, nonlinear frequency parameters of simply supports boundary conditions, a=0.45 m and b=0.30 m.

In Figure 7, AS/3501 and Boron/5505 plates shows similar characteristics. Kev.49/934 plate is different. In Figure 8, three plates show similar characteristics. In Figure 7 and 8, plates show similar properties despite different boundary conditions. In Fig. 9 and 10, the plates show similar behaviors to both boundary conditions. In the first five modes of vibrations, characteristics of plates can be said that similar in all boundary conditions. In all boundary conditions, Kev.49/934 plate shows lowest frequency values and Boron/5505 plate have high frequency values.

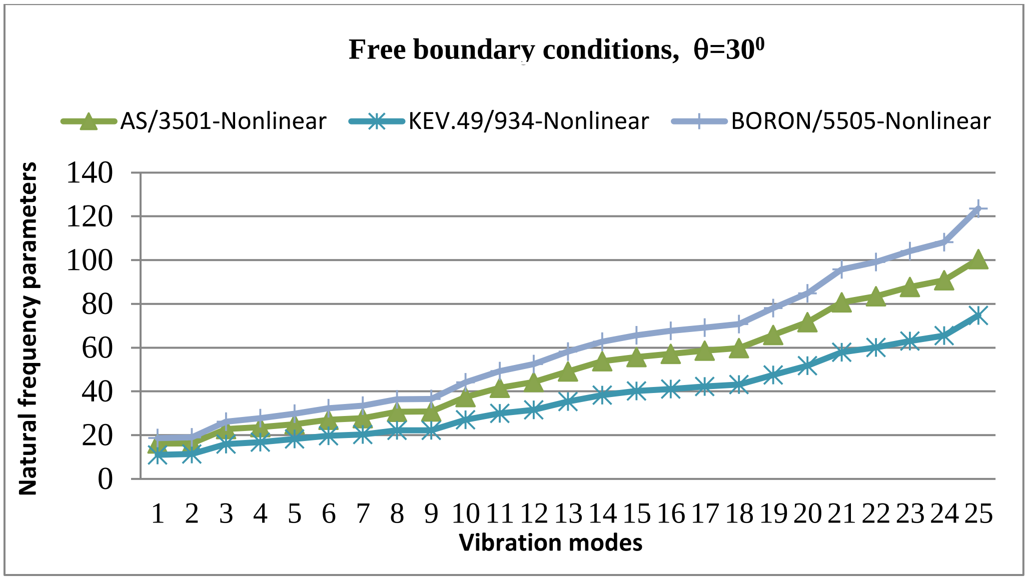

Figure 9. $\theta=30^0$, AS/3501 Graphite/epoxy, Kev.49/934 Kevlar/epoxy, Boron/5505 Boron/epoxy, nonlinear frequency parameters of free boundary conditions, a=0.45 m and b=0.30 m.

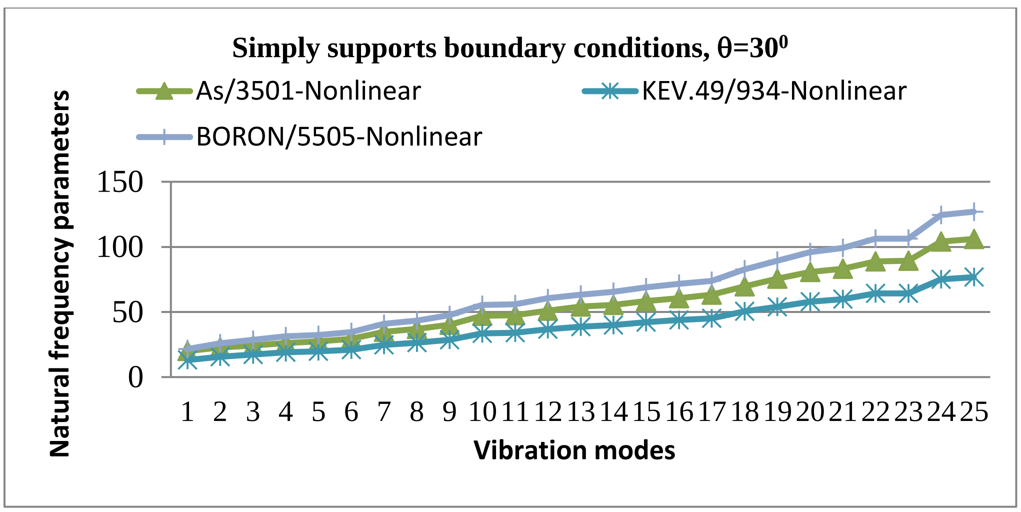

Figure 10. $\theta=30^0$, AS/3501 Graphite/epoxy, Kev.49/934 Kevlar/epoxy, Boron/5505 Boron/epoxy, nonlinear frequency parameters of simply supports boundary conditions, a=0.45 m and b=0.30 m.

5. Discussion

Nonlinear dynamic analysis of laminated composite plates supported by springs is presented in the different boundary conditions, using finite element method. The spring system stiffness matrix $[K_S]$ and nonlinear system stiffness matrix $[K]_{NL}$ was written specifically according to boundary conditions. The boundary conditions affected to vibrations of composite plate. These effects are graphically presented. According to selected composite plates for study, vibrations of plates can be said that similar in the first five modes. In subsequent modes are different. Nonlinear results may be considered in the design of composite plates.

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